

# Temporal-gauge String Field with Open Strings

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## ABSTRACT

A string field theory including open string fields is constructed in the temporal gauge. It consists of string interaction vertices similar to the light-cone gauge string field theory. A slight modification of the definition of the time coordinate is needed because of the existence of the open string end points.

String field theory is considered to be the most promising way to describe physics at the Planck scale nonperturbatively. Recently a new kind of string field theory was formulated [1]. In such a string field theory the nonperturbative aspects of noncritical strings, which are described by matrix model techniques in [2, 3], may be easily deduced. In this string field theory, points on world sheet are parametrized by the geodesic distance [4] on it. This “geodesic distance” may be most easily understood on a dynamically triangulated surface. In the continuum theory, it may be understood as a kind of gauge fixing which we call “temporal gauge” [5] or “proper time gauge” [6]. It is also possible to introduce matter degrees of freedom [7, 8] which have  $c = 1 - 6/m(m + 1)$ .

So far, these string field theories have involved only closed string fields. In this article we will propose a way to introduce open string fields into this “temporal gauge” string field theory for the case  $c = 0$ .

Let us specify the model using via a matrix model. Consider a matrix model action

$$S = \vec{s}^* \vec{s} - \mu \vec{s}^* \phi \vec{s} + \frac{1}{2} \text{tr} \phi^2 - \frac{\lambda}{3} \text{tr} \phi^3. \quad (1)$$

Here  $\phi$  is a Hermitian matrix, and  $\vec{s}$  is a vector which is also a dynamical variable. In this matrix model, open strings appear owing to the first two terms, where we define  $\mu$  to be the tension along the world line of an open string end point. It may be possible to obtain a string field theory by carefully examining the continuum limit of the matrix model [9]. However, here we directly consider the continuum theory and fix the form of the string field Hamiltonian from consistency conditions.

Let us recall the definition of the temporal gauge. Consider a string world sheet with a Euclidean metric. The “time” coordinate of a point on the surface may be defined as the minimum distance from the initial boundaries, where distance is measured by using the metric on the world sheet. As this “time” goes by, strings keep splitting and joining to form world sheets. It should also be noted that disappearance of an infinitesimal string also occurs in this gauge. Each string is parametrized only by its length. In order to describe such processes as splitting and joining, it is convenient to introduce operators which represent annihilation and creation of a string.

$$\begin{aligned} [\Psi(l), \Psi^\dagger(l')] &= \delta(l - l'), \\ [\Phi(l), \Phi^\dagger(l')] &= \delta(l - l'). \end{aligned} \quad (2)$$

Here  $\Psi(l)$  is the operator which annihilates an open string with length  $l$  and  $\Psi^\dagger(l)$  creates an open string with length  $l$ .  $\Phi^\dagger(l)$  creates a closed string with a marked point. This convention takes care of the statistical weights. Of course, other commutators vanish.

The bra and ket vacua ( $\langle 0|$  and  $|0\rangle$ ) satisfy

$$\begin{aligned} \Psi(l)|0\rangle &= 0, & \langle 0|\Psi^\dagger(l) &= 0, \\ \Phi(l)|0\rangle &= 0, & \langle 0|\Phi^\dagger(l) &= 0, \end{aligned} \quad (3)$$

for all  $l$ .

The amplitudes with  $i$  open strings and  $j$  closed strings might be calculated in this formalism as

$$\lim_{D \rightarrow \infty} \langle 0 | e^{-DH} \Psi^\dagger(l_1) \cdots \Psi^\dagger(l_i) \Phi^\dagger(l_{i+1}) \cdots \Phi^\dagger(l_{i+j}) | 0 \rangle, \quad (4)$$

using the string field Hamiltonian  $H$ , which describes the time evolution. It is the peculiarity of the Euclidean theory, that all the strings eventually disappear. This is why we take the limit  $D \rightarrow \infty$ .

The Hamiltonian which we propose here is

$$\begin{aligned} H = & \int dl_1 dl_2 \Phi^\dagger(l_1) \Phi^\dagger(l_2) \Phi(l_1 + l_2) (l_1 + l_2) \\ & + \int dl_1 dl_2 \Psi^\dagger(l_1) \Psi^\dagger(l_2) \Psi(l_1 + l_2) \\ & + 2 \int dl_1 dl_2 \Psi^\dagger(l_1) \Phi^\dagger(l_2) \Psi(l_1 + l_2) l_1 \\ & + g \int dl_1 dl_2 \Phi^\dagger(l_1 + l_2) \Phi(l_1) \Phi(l_2) l_1 l_2 \\ & + 2g \int dl_1 dl_2 \Psi^\dagger(l_1 + l_2) \Phi(l_1) \Psi(l_2) l_1 l_2 \\ & + \sqrt{g} \int dl_1 dl_2 dl_3 dl_4 \Psi^\dagger(l_1 + l_3) \Psi^\dagger(l_2 + l_4) \Psi(l_1 + l_2) \Psi(l_3 + l_4) \\ & + \sqrt{g} \int dl \Psi^\dagger(l) \Phi(l) l \\ & + \int dl \rho(l) \Phi(l) + \int dl \eta(l) \Psi(l) \\ & + 2\sqrt{g} \int dl_1 dl_2 \Psi(l_1) \delta(l_1 + l_2) \Psi(l_2) \\ & + a \left\{ \int dl \Phi^\dagger(l) \Psi(l) + \int dl \Psi^\dagger(l) \left( -\frac{\partial}{\partial l} - m \right) \Psi(l) \right. \\ & \left. + \sqrt{g} \int dl_1 dl_2 \Psi^\dagger(l_1 + l_2) \Psi(l_1) \Psi(l_2) \right\} \end{aligned} \quad (5)$$

All the length variables are integrated from infinitesimal minus value to infinity to make terms with singularity at zero length well defined. This Hamiltonian is derived in the following way.

Considering the processes which might occur during the infinitesimal “proper time” evolution, only terms appearing on the right hand side of eq(5) are possible for the Hamiltonian  $H$ . The statistical weights in the integrals ( e.g.  $l_1 l_2$  in the fourth term of eq.(5) ) are determined in the same way as in the closed string case.  $g$  is the string coupling constant. The power of  $g$  in front of each term is determined by considering how the topology of the surface will change in each process. We know  $g$  has dimension 5 in mass dimension for  $c = 0$  theory. The operator  $\Phi^\dagger(l)$  has dimension 5/2 and  $\Psi^\dagger(l)$  has dimension 3/2. We can see all the terms above have dimension 1/2 except for the terms in the brace. Terms in the brace have dimension 1, where the multiplication

factor  $a$  is arbitrary and has dimension  $-1/2$ . We label “tadpole terms” those terms which have no creation operator. These terms represent the disappearance of a string with infinitesimal length and are the peculiarity of this gauge. We will discuss these terms later and fix the unknown functions  $\rho(l)$  and  $\eta(l)$ . Up to this point, we do not know with what numerical factors all the terms in eq.(5) should appear in the Hamiltonian. The following consistency conditions will fix these factors.

In the string field theory of closed strings [7], the Hamiltonian was written as

$$H_c = \int_0^\infty dl T_c(l) \Phi(l). \quad (6)$$

Here the currents  $T_c(l)$  satisfy the Virasoro algebra:  $[T_c(l), T_c(l')] = gl'(l' - l)/(l' + l)T_c(l + l')$ . The fact that  $T_c(l)$  forms a closed algebra is essential to the consistency of the theory. It means that the equation

$$\langle v_c | T_c(l) = 0. \quad (7)$$

is integrable. Here  $\langle v_c | \equiv \lim_{D \rightarrow \infty} \langle 0 | e^{-DH_c}$ . Eq.(7) is equivalent to the SD equations of the string field theory, which are usually written in the form of  $\langle v_c | T(l) | \text{any state} \rangle = 0$ . One can show that the integrability of eq.(7) is closely related to the residual general coordinate invariance in this gauge [7]. We expect a similar situation in our Hamiltonian. If we include open string fields, the Hamiltonian will appear as

$$\begin{aligned} H = & \int_0^\infty dl T_1(l) \Phi(l) + \int_0^\infty dl T_2(l) \Phi(l) + \dots \\ & + \int_0^\infty dl J_1(l) \Psi(l) + \int_0^\infty dl J_2(l) \Psi(l) + \dots. \end{aligned} \quad (8)$$

The SD equations may be  $\langle v | T_i(l) = \langle v | J_i(l) = 0$ , where  $\langle v | = \lim_{D \rightarrow \infty} e^{-DH}$ . In this case, it is impossible to form a closed algebra because of the four point vertex of open strings appearing in (5). We must relax this requirement and require that  $\langle v | [T_i(l), T_j(l')] = \langle v | [T_i(l), J_j(l')] = \langle v | [J_i(l), J_j(l')] = 0$  can be deduced from  $\langle v | T_i(l) = \langle v | J_i(l) = 0$ . This consistency will fix the numerical factors in  $T$ 's and  $J$ 's.

For the time being, we disregard the tadpole terms. The following three currents form a closed system of constraints.

$$\begin{aligned} T(l) = & \int_0^l dl_1 \Phi^\dagger(l - l_1) \Phi^\dagger(l_1) + g \int_0^\infty dl_1 \Phi^\dagger(l + l_1) \Phi(l_1) \\ & + g \int_0^\infty dl_1 \Psi^\dagger(l + l_1) \Psi(l_1) + \sqrt{g} \Psi^\dagger(l), \end{aligned} \quad (9)$$

$$\begin{aligned} J(l) = & \sqrt{g} \int dl_1 dl_2 dl_3 dl_4 \Psi^\dagger(l_1 + l_3) \Psi^\dagger(l_2 + l_4) \Psi(l_1 + l_2) \delta(l_3 + l_4 - l) \\ & + 2 \int_0^l dl_1 \Psi^\dagger(l - l_1) \Phi^\dagger(l_1) (l - l_1) + g \int_0^\infty dl_1 \Psi^\dagger(l + l_1) \Phi(l_1) l_1 \\ & + \int_0^l dl_1 \Psi^\dagger(l - l_1) \Psi^\dagger(l_1) \end{aligned} \quad (10)$$

$$J'(l) = \Phi^\dagger(l) + \frac{\partial}{\partial l} \Psi^\dagger(l) + \sqrt{g} \int_0^\infty dl_1 \Psi^\dagger(l + l_1) \Psi(l_1). \quad (11)$$

All the terms included in these currents are obtained by stripping off one of the annihilation operators from terms in eq.(5). (Furthermore,  $T(l)$  is divided by  $l$ .) We see each current put on the right of  $\langle v|$  corresponds to the Schwinger-Dyson equation of the matrix model. To calculate commutators of these currents, it is easier to work with the following current.

$$\begin{aligned} J_3(l) &= \left\{ J(l) - \int_0^\infty dx (2x - l) J'(l - x) \Psi^\dagger(l) \right\} / l, \\ &= \sqrt{g} \int_l^\infty dx_1 \int_0^\infty dx_2 \Psi^\dagger(x_1) \Psi^\dagger(x_2) \Psi(x_1 + x_2 - l) \\ &\quad + \int_0^l dl_1 \Psi^\dagger(l - l_1) \Phi^\dagger(l_1) + g \int_0^\infty dl_1 \Psi^\dagger(l + l_1) \Phi(l_1) l_1 + \Psi^\dagger(l) \Psi^\dagger(0). \end{aligned} \quad (12)$$

Using this current the integrability of the constraints becomes

$$[T(l), T(l')] = g(l' - l) T(l + l'), \quad (13)$$

$$[J_3(l), J_3(l')] = \sqrt{g} \int_l^{l'} dx J_3(l + l' - x) \Psi^\dagger(x), \quad (14)$$

$$\begin{aligned} [J_3(l), T(l')] &= g(l' - l) J_3(l + l') \\ &\quad + \sqrt{g} \left\{ \int_l^{l+l'} dx (x - l) + \int_0^{l'} dx (x - l') \right\} J'(l + l' - x) \Psi^\dagger(x), \end{aligned} \quad (15)$$

$$[J_3(l), J'(l')] = -\sqrt{g} J_3(l + l') + \sqrt{g} \int_0^{l'} dx J'(l + l' - x) \Psi^\dagger(x), \quad (16)$$

$$[T(l), J'(l')] = gl' J'(l + l'), \quad (17)$$

$$[J'(l), J'(l')] = 0. \quad (18)$$

In doing the calculation, it is seen that numerical factors are fixed leaving only the freedom to choose the string coupling  $g$ . ( In fact eq(22) fixes the other freedom to rescale  $\Psi(l)$ . )

Now we must include the tadpole terms since the above constraints do not reproduce the tree level amplitude. To the Hamiltonian  $H$  we add the tadpole term for a closed string  $\int dl \rho(l) \Phi(l)$ . The constraint now becomes  $\langle v | \{ l T(l) + \rho(l) \} = 0$ . This equation gives  $\langle v | \{ l T(l) + \rho(l) \} | 0 \rangle = 0$ . Taking the zeroth terms in  $g$  we obtain an equation relating  $\rho(l)$  and the disk amplitude  $f(l)$ . We know the result of the matrix model [10] for  $f(l)$ . In the Laplace transformed form it becomes:  $\tilde{f}(\zeta) = (\zeta - \sqrt{t}/2) \sqrt{\zeta + \sqrt{t}}$ , where  $t$  is the cosmological constant. This result with the constraint equation above fixes the function  $\rho(l)$  [1] as

$$\rho(l) = 3 \delta''(l) - \frac{3}{4} t \delta(l). \quad (19)$$

The equation (13) should be modified to:

$$[l T(l) + \rho(l), l' T(l') + \rho(l')] = gl' \frac{l' - l}{l + l'} \{ (l + l') T(l + l') + \rho(l + l') \}. \quad (20)$$

In order for eq.(20) to be valid,  $ll' \frac{l'-l}{l+l'} \rho(l+l')$  on the right hand side of eq.(20) should vanish. This term seems rather ill-defined because of the singularity at  $l+l'=0$ . Here we deal with the singularity by Laplace transforming it with respect to both  $l$  and  $l'$ .

$$\int_{-\epsilon}^{\infty} dl e^{-l\zeta} \int_{-\epsilon}^{\infty} dl' e^{-l'\zeta} ll' \frac{l'-l}{l+l'} \{3\delta''(l) - \frac{3}{4}t\delta(l)\} = 0. \quad (21)$$

Here the limit  $\epsilon \rightarrow 0$  is taken after integration. This equality is essentially related to the integrability and the general covariance of this theory [7].

The tadpole terms are needed also for open strings. The tree level one open string amplitude  $h(l)$  may be calculated from the disk amplitude for a closed string  $f(l)$  as,

$$h(l) = \int_0^{\infty} dl' e^{-l'm} f(l+l'). \quad (22)$$

Here  $m$  is the tension along the world line of open string end points. The condition  $\langle v|J(l)|0\rangle = 0$  should be modified to include tadpoles. We must add  $-m\Psi^\dagger(l)$  to  $J'(l)$  and to  $J(l)$

$$\eta(l) + 2\sqrt{g} \int_{-\epsilon}^{\infty} dl \delta(l+l_1) \Psi(l_1), \quad (23)$$

where  $\eta(l)$  is fixed as  $-2\delta'(l) - m\delta(l)$ .  $\int dl \delta(l+l_1) \Psi(l_1)$  seems to be zero in Laplace transformed form, but this does not hold if  $\Psi(l)$  is singular at  $l=0$ . In fact this term survives in the commutator  $[J(l), J'(l')]$ . Using eq.(13)-(18), it is easily checked that the equations

$$\langle v|\{lT(l) + \rho(l)\} = 0, \quad (24)$$

$$\langle v|\left\{J(l) + \eta(l) + 2\sqrt{g} \int dl \delta(l+l_1) \Psi(l_1)\right\} = 0, \quad (25)$$

$$\langle v|\{J'(l) - m\Psi^\dagger(l)\} = 0, \quad (26)$$

are integrable using formulas such as eq.(21).

So the Hamiltonian we are seeking is

$$\begin{aligned} H &= \int_0^{\infty} dl \{lT(l) + \rho(l)\} \Phi(l) \\ &+ \int_0^{\infty} dl \left\{J(l) + \eta(l) + 2\sqrt{g} \int dl \delta(l+l_1) \Psi(l_1)\right\} \Psi(l) \\ &+ a \int_0^{\infty} dl \{J'(l) - m\Psi^\dagger(l)\} \Psi(l). \end{aligned} \quad (27)$$

We can see eq.(27) is the same thing as eq.(5). A few comments about this Hamiltonian are in order. It has ambiguities coming from the overall factors of the three currents. The ratio of the first term to the second term in eq(27) must be 1, because  $\int dl_1 dl_2 \Phi^\dagger(l_1) \Phi^\dagger(l_2) \Phi(l_1+l_2) (l_1+l_2)$  in the first term represents the same process as  $\int dl_1 dl_2 l_1 \Psi^\dagger(l_1) \Phi^\dagger(l_2) \Psi(l_1+l_2)$  at the point where the string splits. The remaining ambiguities are absorbed into rescaling of  $a$  and “time”  $D$ . The constant  $a$  has dimension

1/2, so it should not be fixed. The term with  $a$  describes proper time evolution along the open string boundaries. Therefore, if we change  $a$ , we obtain a slightly different definition of our time coordinate. This difference will be seen near the world lines of open string end-points. If  $a \neq 0$ ,  $H$  looks like the light-cone gauge string field theory of Kaku-Kikkawa, since  $H$  has both the vertex in which an open string splits and a vertex in which two open strings merge. However,  $a = 0$  might be the most natural choice because the world sheets have fractal geometry, and the geodesic curves through the surface will be shorter than paths along the boundary. The naive continuum limit of the stochastic quantization of the matrix model as in [11] yields  $a = 0$ .

In this article, open string fields are introduced into the string field theory in the temporal gauge. The form of the Hamiltonian was fixed by considering possible processes and consistency. The string vertices appearing in this formalism look like the vertices of the light-cone gauge string field theory.

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## References

- [1] N. Ishibashi and H. Kawai, Phys. Lett. **B314** (1993) 190.
- [2] E.Br ezin and V.Kazakov, Phys. Lett. **B236** (1990) 144;  
M.Douglas and S.Shenker, Nucl. Phys. **B335** (1990) 635;  
D.Gross and A.Migdal, Phys. Rev. Lett. **64** (1990) 127; Nucl. Phys. **B340** (1990) 333.
- [3] M.Fukuma, H.Kawai and R.Nakayama, Int. J. Mod. Phys. **A6** (1991) 1385;  
R.Dijkgraaf, E.Verlinde and H.Verlinde, Nucl. Phys. **B348** (1991) 435.
- [4] H. Kawai, N. Kawamoto, T. Mogami and Y. Watabiki, Phys. Lett. **B306** (1993) 19.
- [5] M. Fukuma, N. Ishibashi, H. Kawai and M. Ninomiya, Nucl. Phys. **B427** (1994) 139-157.
- [6] R. Nakayama Phys. Lett. **B325** (1994) 347.
- [7] N. Ishibashi and H. Kawai, Phys. Lett. **322** (1994) 67.
- [8] M. Ikehara, N. Ishibashi, H. Kawai, T. Mogami, R. Nakayama and N. Sasakura, to be published in Phys.Rev.D.

- [9] Y. Watabiki, Report No. INS-1017, hep-th@xxx.lanl.gov 9401096.
- [10] E.Br zin, C. Itzykson, G. Parisi and J.B. Zuber, Commun. Math. Phys. **59** (1978) 35.
- [11] A. Jevicki and J. Rodrigues, Nucl. Phys. **B421** (1994) 278.